MATH 63CM HOMEWORK 5

DUE 11:59PM ON SUNDAY, MAY 12

1. Suppose A is a $k \times k$ matrix, B is a $k \times m$ matrix, and C is an $m \times m$ matrix. Prove that

$$\det \begin{bmatrix} A & B \\ \mathbf{0} & C \end{bmatrix} = (\det A)(\det C),$$

(where $\mathbf{0}$ is the $m \times k$ matrix each of whose entries is 0.)

(It follows that the characteristic polynomial of $\begin{bmatrix} A & B \\ \mathbf{0} & C \end{bmatrix}$ is the product of the characteristic polynomials of A and C.)

2. Find all critical points (i.e., all equilibrium points) of the system

$$x' = (x-1)(x+y),$$
$$y' = y - x^2.$$

For each critical point, determine whether it is stable, asymptotically stable, or unstable.

3. Consider the system

$$x' = -2y^3,$$
$$y' = x - 3y^3.$$

(a). Show that $(0,0)$ is a stable equilibrium. [Hint: look for a Lyapunov function of the form $ax^2 + by^4$.] **(b)**. Show that $(0,0)$ is asymptotically stable.

4. Consider the system

$$x' = Ax$$

where A is a real 2×2 matrix with determinant $\det A = \delta$ and $\operatorname{trace}(A) = \tau$.

(a). For which values of δ and τ can you conclude that the origin is a stable equilibrium?

(b). For which values can you conclude that the origin is an asymptotically stable equilibrium?

(c). For which values can you conclude that the origin is an unstable equilibrium?

5. Problem 2.2 in the text.

6. Problem 2.3 in the text.